Основные тригонометрические формулы

$\sin^2\alpha + \cos^2\alpha = 1$ $\quad$ $\sin^2\alpha = 1 - \cos^2\alpha$, $\cos^2\alpha = 1 - \sin^2\alpha$ $\quad$ $1 + \operatorname{tg}^2\alpha = \dfrac{1}{\cos^2\alpha}$ $\quad$ $1 + \operatorname{ctg}^2\alpha = \dfrac{1}{\sin^2\alpha}$

$\dfrac{\sin\alpha}{\cos\alpha} = \operatorname{tg}\alpha = \dfrac{1}{\operatorname{ctg}\alpha}$ $\quad$ $\operatorname{ctg}\alpha = \dfrac{\cos\alpha}{\sin\alpha}$ $\quad$ $\operatorname{tg}\alpha\cdot\operatorname{ctg}\alpha = 1$

$\sec^2\alpha - \operatorname{tg}^2\alpha = 1$ $\quad$ $\operatorname{cosec}^2\alpha - \operatorname{ctg}^2\alpha = 1$ $\quad$ $\operatorname{cosec}\alpha\cdot\sin\alpha = 1$ $\quad$ $\cos\alpha\cdot\sec\alpha = 1$

$$\sin\alpha = \frac{\operatorname{tg}\alpha}{\sqrt{1+\operatorname{tg}^2\alpha}} = \frac{1}{\sqrt{1+\operatorname{ctg}^2\alpha}} \qquad \cos\alpha = \frac{1}{\sqrt{1+\operatorname{tg}^2\alpha}} = \frac{\operatorname{ctg}\alpha}{\sqrt{1+\operatorname{ctg}^2\alpha}}$$

$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}} = \frac{\sqrt{1-\cos^2\alpha}}{\cos\alpha} \qquad \operatorname{ctg}\alpha = \frac{\sqrt{1-\sin^2\alpha}}{\sin\alpha} = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{1}{\operatorname{tg}\alpha}$$

$$A\sin x \pm B\cos x = \sqrt{A^2+B^2}\sin\left(x \pm \operatorname{arctg}\tfrac{B}{A}\right) = \sqrt{A^2+B^2}\cos\left(90^\circ - \left(x \pm \operatorname{arctg}\tfrac{B}{A}\right)\right);$$

$$\underline{A > 0}$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$ $\quad$ $\sin\frac{\alpha}{2} = \sqrt{\dfrac{1-\cos\alpha}{2}}$ $\quad$ $\sin^2\alpha = \dfrac{1-\cos 2\alpha}{2}$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos 2\alpha = 2\cos^2\alpha - 1$ $\quad$ $\cos\frac{\alpha}{2} = \sqrt{\dfrac{1+\cos\alpha}{2}}$ $\quad$ $\cos^2\alpha = \dfrac{1+\cos 2\alpha}{2}$

$\cos 2\alpha = 1 - 2\sin^2\alpha$

$\operatorname{tg} 2\alpha = \dfrac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}$ $\quad$ $\operatorname{tg}\frac{\alpha}{2} = \dfrac{1-\cos\alpha}{\sin\alpha} = \dfrac{\sin\alpha}{1+\cos\alpha}$ $\quad$ $\operatorname{tg}^2\frac{\alpha}{2} = \dfrac{1-\cos\alpha}{1+\cos\alpha}$

$\operatorname{ctg} 2\alpha = \dfrac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha}$ $\quad$ $\operatorname{ctg}\frac{\alpha}{2} = \dfrac{\sin\alpha}{1-\cos\alpha} = \dfrac{1+\cos\alpha}{\sin\alpha}$ $\quad$ $\operatorname{ctg}^2\frac{\alpha}{2} = \dfrac{1+\cos\alpha}{1-\cos\alpha}$

$\operatorname{ctg} 2\alpha = \frac{1}{2}(\operatorname{ctg}\alpha - \operatorname{tg}\alpha)$

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta \qquad \sin\alpha+\sin\beta=2\sin\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta \qquad \sin\alpha-\sin\beta=2\sin\tfrac{\alpha-\beta}{2}\cos\tfrac{\alpha+\beta}{2}$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta \qquad \cos\alpha+\cos\beta=2\cos\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta \qquad \cos\alpha-\cos\beta=-2\sin\tfrac{\alpha+\beta}{2}\sin\tfrac{\alpha-\beta}{2}$$

$$\operatorname{tg}(\alpha+\beta)=\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta} \qquad \operatorname{tg}\alpha+\operatorname{tg}\beta=\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$$

$$\operatorname{tg}(\alpha-\beta)=\frac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{1+\operatorname{tg}\alpha\operatorname{tg}\beta} \qquad \operatorname{tg}\alpha-\operatorname{tg}\beta=\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$$

$$\operatorname{ctg}(\alpha\pm\beta)=\frac{\operatorname{ctg}\alpha\operatorname{ctg}\beta\mp 1}{\operatorname{ctg}\beta\pm\operatorname{ctg}\alpha} \qquad \operatorname{ctg}\alpha\pm\operatorname{ctg}\beta=\frac{\sin(\beta\pm\alpha)}{\sin\alpha\sin\beta}$$

$$2\sin\alpha\sin\beta=(\cos(\alpha-\beta)-\cos(\alpha+\beta))$$

$$2\cos\alpha\cos\beta=(\cos(\alpha-\beta)+\cos(\alpha+\beta))$$

$$2\sin\alpha\cos\beta=(\sin(\alpha-\beta)+\sin(\alpha+\beta))$$

$$\sin\alpha\cos\alpha=\frac{\sin 2\alpha}{2}, \qquad \sin^2\alpha\cos^2\alpha=\frac{\sin^2 2\alpha}{4}$$